

# CONTINUITY & DIFFERENTIABILITY

Assignment 4 Practice by O.P. GUPTA • M. +91-9650350480

Q01. If  $f(x) = \begin{cases} \frac{x^2-1}{x-1}, & \text{when } x \neq 1 \\ k, & \text{when } x = 1 \end{cases}$  is given to be continuous at  $x = 1$ , then the value of  $k$  is

(a) 1 (b) 2 (c) 1 (d) -2

Q02. If  $y = Ae^{5x} + Be^{-5x}$ , then  $\frac{d^2y}{dx^2}$  is equal to

(a) 25 y (b) 5 y (c) -25 y (d) 15 y

Q03. If  $y = \log_e\left(\frac{x^2}{e^2}\right)$ , then  $\frac{d^2y}{dx^2}$ , equals

(a)  $-\frac{1}{x}$  (b)  $-\frac{1}{x^2}$  (c)  $\frac{2}{x^2}$  (d)  $-\frac{2}{x^2}$

Q04. The function  $f: R \rightarrow R$  given by  $f(x) = -|x-1|$  is

(a) continuous as well as differentiable at  $x = 1$   
 (b) not continuous but differentiable at  $x = 1$   
 (c) continuous but not differentiable at  $x = 1$   
 (d) neither continuous nor differentiable at  $x = 1$

Q05. The function  $f(x) = \frac{x-1}{x(x^2-1)}$  is discontinuous at

(a) exactly one point (b) exactly two points  
 (c) exactly three points (d) no point

Q06. The value of  $k$  so that  $f$  defined by  $f(x) = \begin{cases} x^2 \sin\left(\frac{1}{x}\right), & \text{if } x \neq 0 \\ k, & \text{if } x = 0 \end{cases}$  is continuous at  $x = 0$ , is

(a) 0 (b)  $\frac{1}{2}$  (c) 1 (d) 2

Q07. The derivative of  $\log x$  with respect to  $\frac{1}{x}$  is

(a)  $-\frac{1}{x^3}$  (b)  $-\frac{1}{x}$  (c)  $-x$  (d)  $\frac{1}{x}$

Q08. If  $\sec^{-1}\left(\frac{1+x}{1-y}\right) = a$ , then  $\frac{dy}{dx}$  is equal to

(a)  $\frac{x-1}{y-1}$  (b)  $\frac{x-1}{y+1}$  (c)  $\frac{y-1}{x+1}$  (d)  $\frac{y+1}{x-1}$

Q09. If  $x = e^t \sin t$ ,  $y = e^t \cos t$ , then the value of  $\frac{dy}{dx}$  at  $t = \frac{\pi}{4}$  is \_\_\_\_\_.

Q10. The greatest integer function defined by  $f(x) = [x]$ ,  $0 < x < 2$  is not differentiable at  $x =$  \_\_\_\_\_.

Q11. The number of points of discontinuity of  $f(x) = |x| - |x + 1|$  is \_\_\_\_\_.

Q12. If  $f(x) = 2|x| + 3|\sin x| + 6$ , then the right hand derivative of  $f(x)$  at  $x = 0$  is \_\_\_\_\_.

Q13. If  $f(x) = x|x|$ , then  $f'(x) =$  \_\_\_\_\_.

Q14. If  $y = \tan^{-1} x + \cot^{-1} x$ ,  $x \in \mathbb{R}$ , then  $\frac{dy}{dx}$  is equal to \_\_\_\_\_.

Q15. If  $\cos(xy) = k$ , where  $k$  is a constant and  $xy \neq n\pi$ ,  $n \in \mathbb{Z}$ , then  $\frac{dy}{dx}$  is equal to \_\_\_\_\_.

Q16. The value of  $\lambda$ , so that the function  $f$  defined by  $f(x) = \begin{cases} \lambda x, & \text{if } x \leq \pi \\ \cos x, & \text{if } x > \pi \end{cases}$  is continuous at  $x = \pi$  is \_\_\_\_\_.

Q17. If  $f(x) = \sqrt{\frac{\sec x - 1}{\sec x + 1}}$ , find  $f'\left(\frac{\pi}{3}\right)$ .

Q18. If  $y = e^x + e^{-x}$ , then show that  $\frac{dy}{dx} = \sqrt{y^2 - 4}$ .

Q19. Find  $f'(x)$ , if  $f(x) = (\tan x)^{\tan x}$ .

Q20. If  $x = at^2$ ,  $y = 2at$ , then find  $\frac{d^2y}{dx^2}$ .

Q21. Differentiate  $\sec^2(x^2)$  with respect to  $x^2$ .

Q22. If  $y = f(x^2)$  and  $f'(x) = e^{\sqrt{x}}$ , then find  $\frac{dy}{dx}$ .

Q23. Find the value of  $k$ , so that the function  $f(x) = \begin{cases} kx^2 + 5, & \text{if } x \leq 1 \\ 2, & \text{if } x > 1 \end{cases}$  is continuous at  $x = 1$ .

Q24. If  $x = a \sec^3 \theta$ ,  $y = a \tan^3 \theta$ , then find  $\frac{d^2y}{dx^2}$  at  $\theta = \frac{\pi}{4}$ .

Q25. If  $f(x) = \sqrt{\tan \sqrt{x}}$ , then find  $f'\left(\frac{\pi^2}{16}\right)$ .

Q26. Find the relationship between 'a' and 'b' so that the function  $f$  defined by  $f(x) = \begin{cases} ax + 1, & \text{if } x \leq 3 \\ bx + 3, & \text{if } x > 3 \end{cases}$  is continuous at  $x = 3$ .

Q27. Check the differentiability of  $f(x) = |x - 3|$  at  $x = 3$ .

Q28. If  $x = a \sec \theta$ ,  $y = b \tan \theta$  then, find  $\frac{d^2y}{dx^2}$  at  $\theta = \frac{\pi}{6}$ .

Q29. Differentiate  $\sin^2(\sqrt{x})$  with respect to  $x$ .

Q30. If  $y^2 \cos\left(\frac{1}{x}\right) = a^2$ , then find  $\frac{dy}{dx}$ .

Q31. Find the value(s) of  $k$ , so that the function  $f(x) = \begin{cases} \frac{1-\cos kx}{x \sin x} & \text{if } x \neq 0 \\ \frac{1}{2} & \text{if } x = 0 \end{cases}$  is continuous at  $x = 0$ .

Q32. Find the derivative of  $x^{\log x}$  w. r. t.  $\log x$ .

Q33. If the function  $f$  defined as  $f(x) = \begin{cases} \frac{x^2-9}{x-3}, & x \neq 3 \\ k, & x = 3 \end{cases}$  is continuous at  $x = 3$ , find the value of  $k$ .

Q34. If  $x = a \cos \theta$ ,  $y = b \sin \theta$ , then find  $\frac{d^2y}{dx^2}$ .

Q35. Find the differential of  $\sin^2 x$  w. r. t.  $e^{\cos x}$ .

Q36. If  $y = e^{x \sin^2 x} + (\sin x)^x$ , find  $\frac{dy}{dx}$ .

Q37. Prove that the greatest integer function defined by  $f(x) = [x]$ ,  $0 < x < 2$  is not differentiable at  $x = 1$ .

Q38. If  $y = x^3(\cos x)^x + \sin^{-1} \sqrt{x}$ , find  $\frac{dy}{dx}$ .

Q39. If  $x = a \sec \theta$ ,  $y = b \tan \theta$ , then find  $\frac{dy}{dx}$  at  $\theta = \frac{\pi}{3}$ .

Q40. If  $y = \tan^{-1} \left[ \frac{x}{\sqrt{a^2 - x^2}} \right]$ ,  $|x| < a$ , then find  $\frac{dy}{dx}$ .

Q41. If  $y = 3 \cos(\log x) + 4 \sin(\log x)$ , then show that  $x^2 \frac{d^2y}{dx^2} + x \frac{dy}{dx} + y = 0$ .

Q42. If  $x = a \cos \theta + b \sin \theta$ ,  $y = a \sin \theta - b \cos \theta$ , then show that  $\frac{dy}{dx} = -\frac{x}{y}$  and hence show that

$$y^2 \frac{d^2y}{dx^2} - x \frac{dy}{dx} + y = 0.$$

Q43. If  $e^{y-x} = y^x$ , prove that  $\frac{dy}{dx} = \frac{y(1+\log y)}{x \log y}$ .

Q44. If  $x = a(\theta - \sin \theta)$ ,  $y = (1 - \cos \theta)$ ,  $a > 0$ , then find  $\frac{d^2y}{dx^2}$  at  $\theta = \frac{\pi}{3}$ .

Q45. If  $\tan^{-1} \left( \frac{y}{x} \right) = \log \sqrt{x^2 + y^2}$  prove that  $\frac{dy}{dx} = \frac{x+y}{x-y}$ .

Q46. If  $y = e^{x^2 \cos x} + (\cos x)^x$ , then find  $\frac{dy}{dx}$ .

Q47. If  $y = e^{a \cos^{-1} x}$ ,  $-1 < x < 1$ , then show that  $(1-x^2) \frac{d^2y}{dx^2} - x \frac{dy}{dx} - a^2 y = 0$ .

Q48. If  $y = \sin^{-1} \left( \frac{\sqrt{1+x} + \sqrt{1-x}}{2} \right)$ , then show that  $\frac{dy}{dx} = -\frac{1}{2\sqrt{1-x^2}}$ .

Q49. Find the value of  $\frac{dy}{dx}$  at  $\theta = \frac{\pi}{3}$ , if  $x = \cos \theta - \cos 2\theta$ ,  $y = \sin \theta - \sin 2\theta$ .

Q50. Let  $f(x) = x|x|$ , for all  $x \in \mathbb{R}$ , check its differentiability at  $x = 0$ .

Q51. If  $y = (\log x)^x + x^{\log x}$ , then find  $\frac{dy}{dx}$ .

Q52. If  $y = (\cos x)^x + \tan^{-1} \sqrt{x}$ , find  $\frac{dy}{dx}$ .

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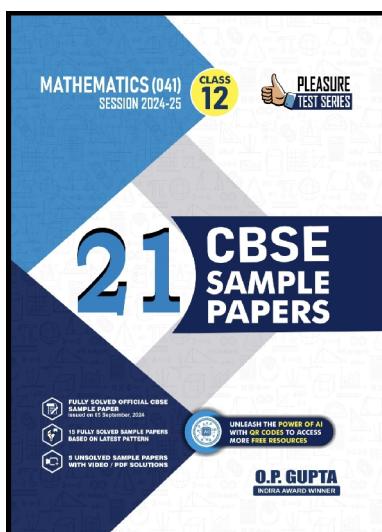
Q53. If  $y = ae^{2x} + be^{-x}$ , then show that  $\frac{d^2y}{dx^2} - \frac{dy}{dx} - 2y = 0$ .

Q54. If  $\sqrt{1-x^2} + \sqrt{1-y^2} = a(x-y)$ , then prove that  $\frac{dy}{dx} = \sqrt{\frac{1-y^2}{1-x^2}}$ .

Q55. If  $x = a(\cos 2\theta + 2\theta \sin 2\theta)$  and  $y = a(\sin 2\theta - 2\theta \cos 2\theta)$ , find  $\frac{d^2y}{dx^2}$  at  $\theta = \frac{\pi}{8}$ .

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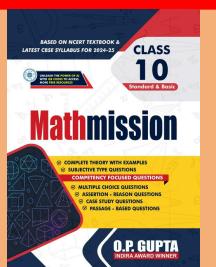
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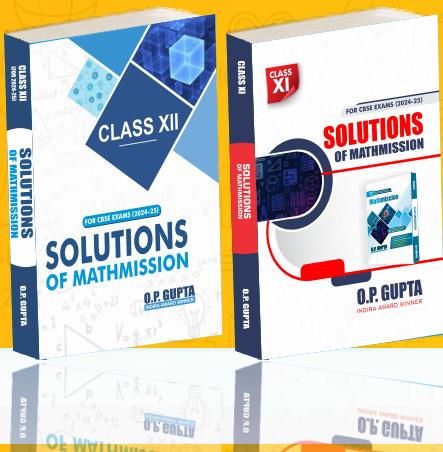


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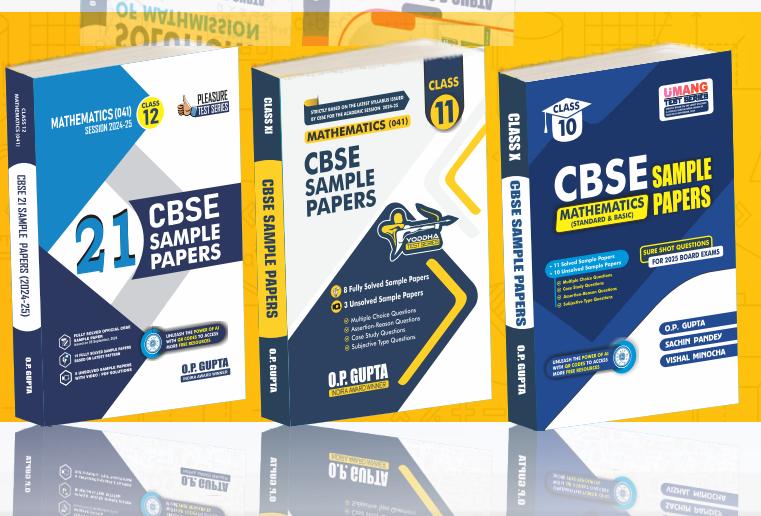
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