

CONTINUITY & DIFFERENTIABILITY

Assignment 4 Practice by O.P. GUPTA • M. +91-9650350480

- Q01. If $f(x) = \begin{cases} \frac{x^2-1}{x-1}, & \text{when } x \neq 1 \\ k, & \text{when } x = 1 \end{cases}$ is given to be continuous at $x = 1$, then the value of k is
- (a) 1 (b) 2 (c) 1 (d) -2
- Q02. If $y = Ae^{5x} + Be^{-5x}$, then $\frac{d^2y}{dx^2}$ is equal to
- (a) 25 y (b) 5 y (c) -25 y (d) 15 y
- Q03. If $y = \log_e \left(\frac{x^2}{e^2} \right)$, then $\frac{d^2y}{dx^2}$, equals
- (a) $-\frac{1}{x}$ (b) $-\frac{1}{x^2}$ (c) $\frac{2}{x^2}$ (d) $-\frac{2}{x^2}$
- Q04. The function $f: \mathbb{R} \rightarrow \mathbb{R}$ given by $f(x) = -|x-1|$ is
- (a) continuous as well as differentiable at $x = 1$
(b) not continuous but differentiable at $x = 1$
(c) continuous but not differentiable at $x = 1$
(d) neither continuous nor differentiable at $x = 1$
- Q05. The function $f(x) = \frac{x-1}{x(x^2-1)}$ is discontinuous at
- (a) exactly one point (b) exactly two points
(c) exactly three points (d) no point
- Q06. The value of k so that f defined by $f(x) = \begin{cases} x^2 \sin\left(\frac{1}{x}\right), & \text{if } x \neq 0 \\ k, & \text{if } x = 0 \end{cases}$ is continuous at $x = 0$, is
- (a) 0 (b) $\frac{1}{2}$ (c) 1 (d) 2
- Q07. The derivative of $\log x$ with respect to $\frac{1}{x}$ is
- (a) $-\frac{1}{x^3}$ (b) $-\frac{1}{x}$ (c) $-x$ (d) $\frac{1}{x}$
- Q08. If $\sec^{-1} \left(\frac{1+x}{1-y} \right) = a$, then $\frac{dy}{dx}$ is equal to
- (a) $\frac{x-1}{y-1}$ (b) $\frac{x-1}{y+1}$ (c) $\frac{y-1}{x+1}$ (d) $\frac{y+1}{x-1}$
- Q09. If $x = e^t \sin t, y = e^t \cos t$, then the value of $\frac{dy}{dx}$ at $t = \frac{\pi}{4}$ is _____.
- Q10. The greatest integer function defined by $f(x) = [x], 0 < x < 2$ is not differentiable at $x = \underline{\hspace{1cm}}$.

- Q11. The number of points of discontinuity of f defined by $f(x) = |x| - |x+1|$ is _____.
- Q12. If $f(x) = 2|x| + 3|\sin x| + 6$, then the right hand derivative of $f(x)$ at $x = 0$ is _____.
- Q13. If $f(x) = x|x|$, then $f'(x) =$ _____.
- Q14. If $y = \tan^{-1} x + \cot^{-1} x$, $x \in \mathbb{R}$, then $\frac{dy}{dx}$ is equal to _____.
- Q15. If $\cos(xy) = k$, where k is a constant and $xy \neq n\pi$, $n \in \mathbb{Z}$, then $\frac{dy}{dx}$ is equal to _____.
- Q16. The value of λ , so that the function f defined by $f(x) = \begin{cases} \lambda x, & \text{if } x \leq \pi \\ \cos x, & \text{if } x > \pi \end{cases}$ is continuous at $x = \pi$ is _____.
- Q17. If $f(x) = \sqrt{\frac{\sec x - 1}{\sec x + 1}}$, find $f'\left(\frac{\pi}{3}\right)$.
- Q18. If $y = e^x + e^{-x}$, then show that $\frac{dy}{dx} = \sqrt{y^2 - 4}$.
- Q19. Find $f'(x)$, if $f(x) = (\tan x)^{\tan x}$.
- Q20. If $x = at^2$, $y = 2at$, then find $\frac{d^2y}{dx^2}$.
- Q21. Differentiate $\sec^2(x^2)$ with respect to x^2 .
- Q22. If $y = f(x^2)$ and $f'(x) = e^{\sqrt{x}}$, then find $\frac{dy}{dx}$.
- Q23. Find the value of k , so that the function $f(x) = \begin{cases} kx^2 + 5, & \text{if } x \leq 1 \\ 2, & \text{if } x > 1 \end{cases}$ is continuous at $x = 1$.
- Q24. If $x = a \sec^3 \theta$, $y = a \tan^3 \theta$, then find $\frac{d^2y}{dx^2}$ at $\theta = \frac{\pi}{4}$.
- Q25. If $f(x) = \sqrt{\tan \sqrt{x}}$, then find $f'\left(\frac{\pi^2}{16}\right)$.
- Q26. Find the relationship between 'a' and 'b' so that the function f defined by $f(x) = \begin{cases} ax + 1, & \text{if } x \leq 3 \\ bx + 3, & \text{if } x > 3 \end{cases}$ is continuous at $x = 3$.
- Q27. Check the differentiability of $f(x) = |x - 3|$ at $x = 3$.
- Q28. If $x = a \sec \theta$, $y = b \tan \theta$ then, find $\frac{d^2y}{dx^2}$ at $\theta = \frac{\pi}{6}$.
- Q29. Differentiate $\sin^2(\sqrt{x})$ with respect to x .
- Q30. If $y^2 \cos\left(\frac{1}{x}\right) = a^2$, then find $\frac{dy}{dx}$.

Q31. Find the value(s) of k , so that the function $f(x) = \begin{cases} \frac{1 - \cos kx}{x \sin x} & \text{if } x \neq 0 \\ \frac{1}{2} & \text{if } x = 0 \end{cases}$ is continuous at $x = 0$.

Q32. Find the derivative of $x^{\log x}$ w. r. t. $\log x$.

Q33. If the function f defined as $f(x) = \begin{cases} \frac{x^2 - 9}{x - 3}, & x \neq 3 \\ k, & x = 3 \end{cases}$ is continuous at $x = 3$, find the value of k .

Q34. If $x = a \cos \theta$, $y = b \sin \theta$, then find $\frac{d^2 y}{dx^2}$.

Q35. Find the differential of $\sin^2 x$ w. r. t. $e^{\cos x}$.

Q36. If $y = e^{x \sin^2 x} + (\sin x)^x$, find $\frac{dy}{dx}$.

Q37. Prove that the greatest integer function defined by $f(x) = [x]$, $0 < x < 2$ is not differentiable at $x = 1$.

Q38. If $y = x^3 (\cos x)^x + \sin^{-1} \sqrt{x}$, find $\frac{dy}{dx}$.

Q39. If $x = a \sec \theta$, $y = b \tan \theta$, then find $\frac{dy}{dx}$ at $\theta = \frac{\pi}{3}$.

Q40. If $y = \tan^{-1} \left[\frac{x}{\sqrt{a^2 - x^2}} \right]$, $|x| < a$, then find $\frac{dy}{dx}$.

Q41. If $y = 3 \cos(\log x) + 4 \sin(\log x)$, then show that $x^2 \frac{d^2 y}{dx^2} + x \frac{dy}{dx} + y = 0$.

Q42. If $x = a \cos \theta + b \sin \theta$, $y = a \sin \theta - b \cos \theta$, then show that $\frac{dy}{dx} = -\frac{x}{y}$ and hence show that

$$y^2 \frac{d^2 y}{dx^2} - x \frac{dy}{dx} + y = 0.$$

Q43. If $e^{y-x} = y^x$, prove that $\frac{dy}{dx} = \frac{y(1 + \log y)}{x \log y}$.

Q44. If $x = a(\theta - \sin \theta)$, $y = (1 - \cos \theta)$, $a > 0$, then find $\frac{d^2 y}{dx^2}$ at $\theta = \frac{\pi}{3}$.

Q45. If $\tan^{-1} \left(\frac{y}{x} \right) = \log \sqrt{x^2 + y^2}$ prove that $\frac{dy}{dx} = \frac{x + y}{x - y}$.

Q46. If $y = e^{x^2 \cos x} + (\cos x)^x$, then find $\frac{dy}{dx}$.

Q47. If $y = e^{a \cos^{-1} x}$, $-1 < x < 1$, then show that $(1 - x^2) \frac{d^2 y}{dx^2} - x \frac{dy}{dx} - a^2 y = 0$.

Q48. If $y = \sin^{-1} \left(\frac{\sqrt{1+x} + \sqrt{1-x}}{2} \right)$, then show that $\frac{dy}{dx} = -\frac{1}{2\sqrt{1-x^2}}$.

Q49. Find the value of $\frac{dy}{dx}$ at $\theta = \frac{\pi}{3}$, if $x = \cos \theta - \cos 2\theta$, $y = \sin \theta - \sin 2\theta$.

Q50. Let $f(x) = x|x|$, for all $x \in \mathbb{R}$, check its differentiability at $x = 0$.

Q51. If $y = (\log x)^x + x^{\log x}$, then find $\frac{dy}{dx}$.

Q52. If $y = (\cos x)^x + \tan^{-1} \sqrt{x}$, find $\frac{dy}{dx}$.

Q53. If $y = ae^{2x} + be^{-x}$, then show that $\frac{d^2y}{dx^2} - \frac{dy}{dx} - 2y = 0$.

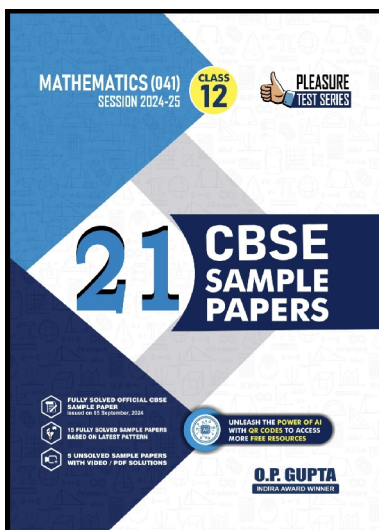
Q54. If $\sqrt{1-x^2} + \sqrt{1-y^2} = a(x-y)$, then prove that $\frac{dy}{dx} = \sqrt{\frac{1-y^2}{1-x^2}}$.

Q55. If $x = a(\cos 2\theta + 2\theta \sin 2\theta)$ and $y = a(\sin 2\theta - 2\theta \cos 2\theta)$, find $\frac{d^2y}{dx^2}$ at $\theta = \frac{\pi}{8}$.

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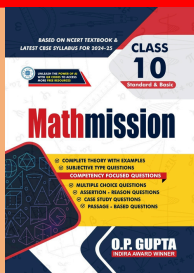
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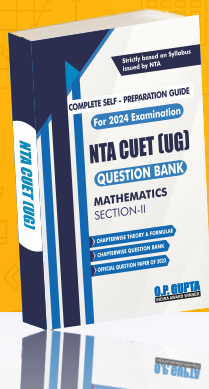


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